For Students

Stop having sex – the world is overpopulated!

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What's the deal with this so-called population *problem*?

The phenomenon that the human species reproduces itself is quite natural. But, when the there is no longer enough resources in a species' natural habitat to sustain the amount of individuals; it is equally natural that some individuals of that species will find it difficult to survive. When that happens we call that habitat *overpopulated.* You have probably heard that there are matters concerning the growth of the world population that call upon our attention. During the last 200 years the world population has sextupled, and, as you will see, in some continents the *growth rate* has been even higher. In the following you are going to discuss some issues related to population growth. In order to do that you will need to familiarize yourself with some basic mathematical tools. In particular you are going to discuss which factors have an influence on the population size, and what, if overpopulation is a threat, we should do about it.

> The ***growth rate*** of a country's population is the rate at which the number of people in that country increases. Usually the growth rate refers to the changes in population size over a specific period. Usually the *unit* for the growth rate is percentage of the population at the beginning of the period in question. (Note that the growth rate can be negative as well.)

Background:

The interest in the development of the population size began during the 18th century. At that time focus was directed at the ratio between a population size and the consumption of natural resources. In 1798, the British economist Thomas R. Malthus (1766-1834), published the book *Essay on the Principle on Population*, in which he presented a mathematical ***model*** for the growth of a population. If you haven't yet read the handout called "Mathematical models", now is a good time to do so, and familiarize yourself with the notion of a model.

Picture 1: Thomas R. Malthus (1766-1834)

Assignment 1: Construct a simple verbal model of population growth within your country. Think about which factors influence the population size. And explain in words how these factors together influence the population size.

The problem:

In his book, Malthus wrote that the population grows exponentially whereas the supply of natural resources grows

linearly. According to Malthus, this poses a serious problem: he thought that the world population grows so fast that after one century a shortage of natural resources would result in famine.

On the basis of his model, Malthus recommended that the birthrate should be reduced by delaying matrimony. Malthus feared, however, that such an initiative would result in the deterioration of morality because it would lead to more premarital sex. So Malthus thought that one way to decrease the growth rate was people stop having sex. But he also recognized that this solution would be difficult to persuade people of.

Text

As we now know, Malthus' forecasts were not exactly right for *his* part of the world. However, overpopulation is a threat in some areas of the world. Before we move on, let us look more closely at Malthus' model.

Malthus' model can be given a symbolic description in terms of this formula:

$$P_t = P_o \cdot (1 + r)^t$$

Here P_t is the population size at time t, P_o is the initial population size, r is the annual growth rate (e.g. if the annual population growth is 1.5% the growth rate would be r=0.015), and t is the number of years after the start time.

Assignment 2: Make a graphical model from Malthus' model. Does the result surprise you? Why/why not? Discuss how to make a verbal model out of Malthus' symbolic model. How does this model compare to the model, which you constructed in the last assignment?

Here are some data of the population in England and Wales a short time after Malthus made his model:

Year	1801	1811	1821	1831	1841	1851	1861	1871	1881	1891	1901	1911
Mio.	8.89	10.16	12.00	13.9	15.91	17.93	20.07	22.71	25.97	29.00	32.53	36.07

Table 1: The population in England and Wales during 1801 and 1911.

Assignment 3: Can you find a pattern in the data in Table 1 and describe it? Try to construct a graph from this data. Explain the kind of growth you witness. See the hint if you are in doubt.

Assignment 4: Explain the idea behind the hint to the last question.

Assignment 5: Determine whether the population growth in England and Wales between 1801 and 1911 is linear or exponential. Construct a symbolic model

Hint: A very simple method to approach such a table is to determine the difference or quotient between two, on each other following, columns. If the *difference* between two, on each other following, columns is constant for each column, then the population growth is *linear*. If the *quotient* between two, on each other following, columns is constant for each column, then the population growth is *exponential*.

of the population growth in England and Wales between 1801 and 1911. What does your discovery mean? And why does the population growth behave in this way? Do you think that the population continued to grow in this fashion?

Overpopulation – a problem for the third world only?

Assignment 6: Discuss how overpopulation in an area influences

- The natural resources in that area
- The environment in that area
- The social conditions in that area
- The economic conditions in that area
- These conditions in other areas

Assignment 7: Take a closer look on Table 2. Can you find a pattern for the world population and for the individual continents? Construct a verbal, graphical and symbolic model of the development of the world population from 1750 to 2005. Do the same for the continents Europe, Asia and Africa.

Estimated world population at various dates, in thousands

Year	World	Africa	Asia	Europe	Latin America	Northern America*	Oceania
-8000	5 000**						
-1000	50 000						
-500	100 000						
1	200,000+						
1000	310 000						
1750	791 000	106 000	502 000	163 000	16 000	2 000	2 000
1800	978 000	107 000	635 000	203 000	24 000	7 000	2 000
1850	1 262 000	111 000	809 000	276 000	38 000	26 000	2 000
1900	1 650 000	133 000	947 000	408 000	74 000	82 000	6 000
1950	2 518 629	221 214	1 398 488	547 403	167 097	171 616	12 812
1955	2 755 823	246 746	1 541 947	575 184	190 797	186 884	14 265
1960	2 981 659	277 398	1 674 336	601 401	209 303	204 152	15 888
1965	3 334 874	313 744	1 899 424	634 026	250 452	219 570	17 657
1970	3 692 492	357 283	2 143 118	655 855	284 856	231 937	19 443
1975	4 068 109	408 160	2 397 512	675 542	321 906	243 425	21 564
1980	4 434 682	469 618	2 632 335	692 431	361 401	256 068	22 828
1985	4 830 979	541 814	2 887 552	706 009	401 469	269 456	24 678
1990	5 263 593	622 443	3 167 807	721 582	441 525	283 549	26 687
1995	5 674 380	707 462	3 430 052	727 405	481 099	299 438	28 924
2000	6 070 581	795 671	3 679 737	727 986	520 229	315 915	31 043
2005	6 453 628	887 964	3 917 508	724 722	558 281	332 156	32 998**

Table 2: Estimated world population at various dates, in thousands. Taken from http://en.wikipedia.org/wiki/World_population. (* *Northern America indicates USA and Canada*; ** *This figure is disputed*)

Assignment 8: Use your *verbal* model of the development of the world population from the previous assignment to predict the world population in 2050, 2100 and 2150. Give the same prediction for Europe, Asia and Africa by using your verbal models. Now do this with your graphical and symbolic models from the previous assignment as well. Compare the predictions drawn from each type of model. Which type of model do you feel most comfortable to use?

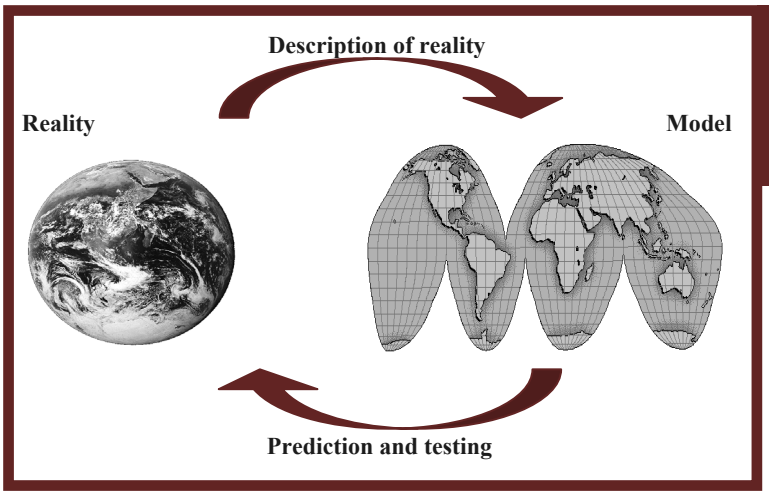
Assignment 9: According to your predictions, how big a percentage of the total world population would the populations of Europe, Asia and Africa, respectively, stand for in 2050, 2100, and 2150. Compare these figures with the percentage distribution of 2005. What do you think when you see these figures?

For many years the government in the People's Republic of China has recognized that it has to deal with the problem of overpopulation. And it has taken drastic measures. As you probably know, China has a ***one-child policy*** according to which families who live in urban areas have to pay a compensation fee to the government if they have more than one child. This of course is an effective way to persuade people to have fewer children, and, in theory, this at least forestalls the overpopulation problem. So from a simple mathematical perspective the one-child policy is a solution. But the one-child policy is quite problematic if further factors are taken into account. One notable problem is the *one-two-four* problem: When each couple gives birth to only one child, that child will eventually have to provide for its two parents and four grandparents. So, from a more elaborate perspective the one-child policy carries its own problems.

Assignment 10: Do you think that the one-child policy is ethically correct? Is there a better way of solving the over-population problem in areas such as China? And what would the criteria be, for a good solution? That it works? That it takes account of the people's needs and rights? Or is overpopulation not as big a threat as it appears?

Mathematical models

An important reason to learn mathematics is that in doing so one acquires methods to solve real-life problems. The problem areas that are described using mathematics are often immensely complex. Therefore it can be necessary to simplify and idealize the situation. This is why mathematical descriptions of real-life situations are called *mathematical models* of reality.



There are other models than mathematical models. A world map is also a model of reality.

The construction and application of a mathematical model is usually a process in which the individual steps must be repeated. The first version of a model typically yields some predictions on the problem at hand. These predictions can be tested against the course of reality. This test may, in turn, lead to improvements on a second version of the model and thereby to new predictions, which, again, can be tested. Repeating this process often leads to very precise predictions on real-life situations.

The construction of a mathematical model often includes one of the following description methods.

1. One can give a *numerical* description. Here one would usually arrange a series of data in a table which describes a specific development.
2. One can give a *symbolic* description. Here one would use mathematical symbols and expressions to describe a specific development
3. One can give a *graphical* description. Here one would describe a situation by means of a graph in a system of coordinates.

Height	**160**	**171**	**172**
Weight	**66**	**68**	**75**

Numerical method

$$P = \alpha \cdot t + \beta$$

Symbolic method

Graphical method

Verbal Models

Before the construction of a mathematical model of a situation it is a good idea to describe that situation in words. In that way it can be easier to see how the mathematical model can be constructed. This is why some mathematical models begin as *rules of*

thumb – a simple description with words of a situation in real-life. A good example of a rule of thumb – or a verbal model – is:

"Your height at age 2 is half your eventual height as an adult"

Having constructed such a verbal model it is possible to continue to construct a mathematical model on the basis of the verbal model. Try to construct other types of models, such as graphic and symbolic models on the basis of this rule of thumb.