# What is a fair insurance premium? 

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Subject: Mathematical modeling of the alternative ways to determine car insurance premiums
Grade level: 10-12
Curriculum content: Mathematics
Kind of activity: Discussions, thought eliciting activity, model eliciting activity
Anticipated time: 4-12 lessons

## Task description

Have you considered buying a car in the near future? That could be an expensive affair with respect to auto insurance. Up until now most insurance companies have had only national statistics at their disposal when determining the yearly insurance premium (the amount the consumer has to pay, in order for his or her car to be insured). And these statistics are not promising: For 18-19 years olds the risk of being involved in a serious accident is 7 times as high as that of your parents. If you are between 20 and 24 years old, the risk is 3 times as high as that of your parents. And surely, if the risk that you will be involved in an accident is increased, there is an increased risk that you will make insurance claims to your insurance company.

But is this a fair way to determine the yearly premium? What about those young people who actually are good drivers? And what about those 40 year olds who drive recklessly? Even many insurance companies agree that this way of calculating the yearly premium is not fair. And some have started to use innovative technology instead of just national statistics.

In table 1 you will find some data on ten drivers including their individual yearly premiums.

Assignment 1: Can you find a pattern in Table 1? Which mathematical tools (graphs, table, equations etc.) did you use to identify a pattern, and why did you use these tools? Which three factors (i.e. which ${ }^{2}$ frie nivesitat (19) Serin $\| \mid$ || ${ }^{10}$ ( UNIVERSITYoTARTU
columns) seem to have a decisive effect on the insurance premium? Why do you think that these factors seem to be important? Give an estimate of the relative weight of these factors.

The insurance company Pay As You Drive, for one wanted to be fair towards their costumers. And the director has asked himself, "what if the premium is determined by how you actually drive, and not by how people with your characteristics drive in general?" In a test period of one month,

Discuss in your group and with your teacher what you know about how a GPS navigation device works. How does it capture a location? How can it be used to determine speed? Pay As You Drive asked the ten costumers mentioned above to install a GPS spy device in their car. This device is, on the one hand, a normal GPS navigation device which can capture location and speed data of a car. On the other hand, the device is connected to network through a 3G broadband modem, and the device is able to transmit data to a server at Pay As You Drive's analysis office.

In Tables 2 and 3 you find some of the data captured by the GPS spy device.

Assignment 2: As you can see the company has, on before hand, categorized some stretches of road as "very dangerous", "dangerous" and "less dangerous". Which factors would you focus on in such a classification? Find your own way to make it easy to classify a given stretch of road on the grounds of data about that road respective to these factors.

Assignment 3: Choose from all the factors found in the three tables those which you would focus on if you were to help Pay As You Drive to determine individual premiums for the ten costumers. Why did you choose these factors? And should they have an equal relative weight.

Assignment 4: Construct a model which takes into account the factors you identified in the last assignment. You can use whatever modeling tool you like - i.e. you can draw graphs, make tables, make equations etc. The model should be a tool for determining a fair individual premium for the costumers.

Assignment 5: Discuss benefits and drawbacks of using a GPS spy device. Are there other benefits than determining a fair insurance premium? Is it in fact more fair to determine the premium in this way?


## Tables

| Person | Age | Years with driver's license | Years with own car | Type of car | Car produced in... | Number of claims in the last 3 years | Miles driven per day | $\begin{aligned} & \text { Yearly } \\ & \text { premium } \\ & \text { (in } € \text { ) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Andrew | 59 | 41 | 41 | Large | 1999 | 1 | 203 | 730 |
| Beth | 26 | 1 | 1 | Small | 1998 | 0 | 50 | 2068 |
| Christina | 42 | 23 | 12 | Medium | 1987 | 3 | 124 | 2000 |
| Dennis | 19 | 1 | 1 | Fast | 1984 | 0 | 88 | 2619 |
| Elaine | 30 | 12 | 7 | Medium | 2007 | 0 | 10 | 784 |
| Frank | 32 | 14 | 11 | Large | 2001 | 1 | 388 | 1176 |
| George | 78 | 60 | 60 | Small | 1992 | 0 | 9 | 608 |
| Hillary | 19 | 1 | 1 | Small | 2000 | 1 | 29 | 2500 |
| Ian | 23 | 5 | 5 | Small | 1998 | 2 | 153 | 2654 |
| Jeremiah | 27 | 2 | 1 | Fast | 2004 | 0 | 98 | 1135 |

Table 1 - General information.

| Person | Driving in urban areas <br> (max. 40 mph ) |  | Driving on highway (max. 80 mph ) |  | Driving on freeway (max. 100 mph ) |  | Miles driven on very dangerous roads | Miles driven on dangerous roads | Miles driven on less dangerous roads |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | miles | Top speed | miles | Top speed | miles | Top speed |  |  |  |
| Andrew | 50 | 39 mph | 136 | 89 mph | 17 | 112 mph | 20 | 93 | 110 |
| Beth | 2 | 38 mph | 48 | 76 mph | - | - | 48 | - | 2 |
| Christina | 70 | 43 mph | 12 | 84 mph | 42 | 103 mph | - | 12 | 112 |
| Dennis | - | - | 88 | 112 mph | - | - | 68 | 20 | - |
| Elaine | 10 | 32 mph | - | - | - | - | - | - | 10 |
| Frank | 70 | 54 mph | 68 | 90 mph | 250 | 110 mph | 89 | 130 | 169 |
| George | 9 | 24 mph | - | - | - | - | - | - | 9 |
| Hillary | 4 | 39 mph | 25 | 89 mph | - | - | 12 | 12 | 5 |
| Ian | - | - | 153 | 98 mph | - | - | 60 | 73 | 20 |
| Jeremiah | 12 | 44 mph | 12 | 97 mph | 74 | 115 mph | 16 | 30 | 52 |

Table 2 - Driving habits as revealed by the GPS spy-device.

| Person |  | Drives primarily... |
| :---: | :---: | :---: |
| Andrew | - Parks 8 hours on a private company parking lot. <br> - Drives two times a day through three dangerous intersections in each of which on average 18 serious accidents happen each year. | during rush hours |
| Beth | - Drives on a very dangerous rural highway where he has to overtake a number of agricultural machines. | during off-peak hours |
| Christina | - Parks 9 hours on a public parking lot with room for 2000 cars where on average 1 minor accident takes place per day and where on average 5 cars are stolen each year. | during rush hours |
| Dennis | - Drives on a very dangerous rural highway where he has to overtake a number of agricultural machines. | during off-peak hours |
| Elaine | - Parks 8 hours on a private company parking lot. <br> - Turns right at three intersections where there is an increased risk of right-turn accidents involving bicycle riders. | during rush hours |
| Frank | - Drives on a stretch of freeway where a lot of solo accidents happen. <br> - Drives on a very dangerous rural highway where he has to overtake a number of agricultural machines. <br> - Drives two times a day through eight dangerous intersections in which on average 5 to 7 serious accidents happen each year. <br> - Parks 8 hours on a private company parking lot. | during rush hours |
| George |  | during off-peak hours |
| Hillary | - Drives on a very dangerous rural highway where he has to overtake a number of agricultural machines. | during rush hours |
| Ian | - Drives on a very dangerous rural highway where he has to overtake a number of agricultural machines. <br> - Drives two times a day through four dangerous intersections in which on average 9 to 11 serious accidents happen each year. | during off-peak hours |
| Jeremiah | - Drives two times a day through a dangerous intersection in which on average 20 serious accidents happen each year. | during rush hours |

Table 3 - Additional information revealed by the GPS spy-device.
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