For Teacher

It wouldn't do any harm to drive 60 km/h in a city instead of 50 km/h would it?

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Subject: Mathematical modeling of the alternative ways to determine car insurance premiums

Grade level: 10-12

Curriculum content: Mathematics

Kind of activity: Discussions, thought eliciting activity, model eliciting activity

Anticipated time: 4-5 lessons

These activities enable students to strengthen their competencies of

1) Mathematical modeling, hereunder
    a. Discussing specific mathematical models
    b. Understanding and communicating about general aspects of mathematical modeling
2) Arguing from mathematics, hereunder
    a. Predicting from models
3) Data handling, hereunder
    a. Graphing data
    b. Tabularizing data
4) Evaluating graphs and functions
5) Identifying dependence relations between phenomena and translate those into dependence relations between variables

Learning outcomes per activity

Having engaged with **assignment 1**, students are expected to be able to

- Verbalize the information gathered from a graph.
- Apply this information so as to infer information about the deceleration of a car.
- Have an understanding of finding the distance traveled by a car, and a car's braking distance, using graphical means only.

Having engaged with **assignment 2**, students are expected to be able to

- Applying graphical tools and equations to solve problems. (using a count of triangles to determine the relative size of two triangles)
- Understand why braking distance is not *linearly* proportional to initial velocity, but that braking distance quadruples as velocity doubles.
- Construct a means for predicting the relative size of a braking distance to another for any relative change in velocity.
- Establish an individual opinion on how the velocity of cars in light of the mathematical information.

Having engaged with **assignment 3**, students are expected to be able to

- Identify the braking ability of a car as the slope of the declining line in picture 1.
- Understand that relative braking distance is unaffected by braking ability.

Having engaged with **assignment 4**, students are expected to be able to

- Apply a number of modeling tools so as to mathematize commonly known phenomena.
- Use these tools to make predictions of the behavior of braking cars.

Having engaged with **assignment 5 and 6**, students are expected to be able to

- Be sensitive to the fact that a model is an approximation, and that some factors should be included because they are crucial.
- Incorporate reaction time in their model and make new prediction (in that sense, understand that modeling is an iterative process)

Proposed plan

- Groups should be established at the beginning of the first lesson.
- Midway through it may be a good idea to summarize and let each group present their considerations in front of the whole class.
- Ending with a similar presentation round can be beneficial.

Suggested teaching strategy

1. These open-ended activities afford opportunities for students to be reflective and critical of the mathematical models upon which decisions about well-known aspects of everyday life rest.

2. Students should work in groups of 3-5, and everybody should be encouraged to participate actively in the discussions. Students can be allowed to reveal to the

teacher how they think about, and put to use, mathematics in specific situations, by being prompted to communicate and represent their ideas to other group members and other groups.

3. The teacher should play the role of an interpreter, by listening to students' proposed solutions and endeavoring to refine students' responses. The nature of the activities allows students to evaluate their proposed solutions themselves. It is not, as such, important that students find a "correct" solution. Rather, students should strengthen their deeper understanding of how to argue from mathematics and what it means to model complex and dynamics processes in society.

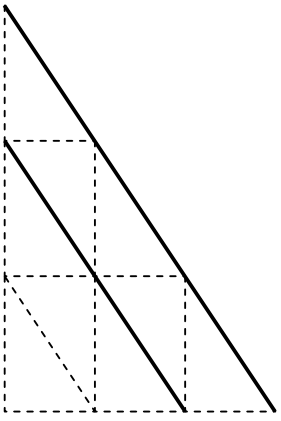
4. The teacher should use this interpretive role as a background for facilitating, stabilizing and refining students' usage of scientific, technological and mathematical concepts.

5. The students' work in groups should be backed up with 2 or 3 summative discussion-/presentation-sessions in which the entire class participates. These sessions allows the teacher to fine-tune students' usage of mathematical concepts.

6. If it is required the teacher may preface the activities by discussing what it means to construct mathematical models. Hereunder
    a. Introducing the different modeling tools (graphs, tables, equations, verbalizations, rules of thumb, etc.)
    b. Discussing the power to predict from models, and what it means to argue from a mathematical model)

7. In assignment 1, students should be encouraged to verbalize their thoughts about the graph in picture 1. This verbalization should begin in the wordings of the students, but can be refined by the teacher during the verbalization process. From this verbalization students should be encouraged to say what they can infer about the deceleration and braking distance of, and total distance traveled by, the car.

8. In assignment 2, students should be encouraged to approach the problem with graphical tools (e.g. triangles) – like the one used in this illustration. But in the end of this assignment students will have to calculate the ratios the general case.



9. In assignment 3, students should be encouraged to verbalize how changes in braking ability affect the outlook of the graph in picture 1. They should be encouraged to reflect on how this matches with their intuition.

10. In assignment 4, 5 and 6, students should be encouraged to construct a model of the situation described and in a number of steps make predictions, refine the model in light of new information, and make new predictions. Encourage them to verbalize their understanding of the process they are going through as they are taking these steps.